

Multirate filter as well as display system and mobile telephone comprising said multirate filter

The invention relates to a multirate filter as well as a display system and a mobile phone comprising a multirate filter.

Digital filters find widespread use in audio and video processing, for example in mobile phones, set top boxes, digital television sets, and other consumer or professional products. Symmetric filters form an important class, because of their linear phase property and the possibility to exploit this symmetry to simplify the architecture of the filter and therewith reduce costs.

In particular, multirate filters are used in applications where the output signal and the input signal of the filter should have mutually different sample rates. Such filters are applied for example in image processing to effect a scaling of a digitally encoded image. One of the most important concepts in multirate filtering is the polyphase decomposition and the closely related polyphase structure. This concept allows for very efficient implementations both in hardware and software of interpolating and decimating filters.

A straightforward application of the polyphase decomposition however usually introduces asymmetric polyphase components, which substantially reduces the efficiency of the implementation

It is a purpose of the invention to provide an architecture for a multirate filter wherein the symmetry of its components is recovered. For that purpose the multirate filter according to the invention has a construction as defined in claim 1. It has been recognized by the inventors that a symmetric multirate filter may be constructed from an input unit, a filter unit and an output unit, wherein the filter unit has symmetric modules derived from the polyphase components of the multirate filter provided that the modules are provided in pairs having transfer functions  $H_0(z)$  and  $H_1(z)$ , which are derived from a basic transfer function  $H_B(z)$  as follows. The first one  $H_0(z)$  of the transfer functions is based on the sum of the basic transfer function  $H_B(z)$  and its mirrored version:

$$H_0(z) = c_0(H_B(z) + M_{\alpha, \psi} H_B(z)), \text{ and}$$

the second one  $H_1(z)$  is based on the difference of the basic transfer function  $H_B(z)$  and its mirrored version:

$$H_1(z) = c_1(H_B(z) - M_{\alpha,\psi}H_B(z)).$$

The mirror operation  $M_{\alpha,\psi}$  on the basic transfer function  $H_B(z)$  is defined as:

$$M_{\alpha,\psi}(H_B(z)) = \alpha z^{-2\psi} H_B^*(z^{-1}), \text{ and wherein}$$

$$H_B^*(z) = \sum h_b^*[m]z^{-m}, H_B(z) \text{ being the } z\text{-transform of } h_b[m]$$

- 5 Therein the value  $2\psi$  is an integer number, selected from  $\mathbb{Z}$ , and the value  $\alpha$  is an element from the set  $\mathbb{C}$  of complex numbers.

More in particular it has been discovered by the inventors that the asymmetric components of the polyphase filter can be redesigned into one of the embodiments described in claims 2 and 5, or a combination thereof.

- 10 In the embodiment of claim 2 the basic transfer function  $H_B(z)$  is a polyphase component of the multirate filter. The embodiment of claim 2 has two sub-embodiments as defined in claim 3 and claim 4. In the subembodiment of claim 3 the input unit comprises a combination unit as defined in claim 1, and claim 4 the output unit comprises such a combination unit. In the embodiment of claim 5 the basic transfer function  $H_B(z)$  is a linear  
15 function of two polyphase components is of the multirate filter.

- The invention further relates to a method of designing a multirate filter according to the invention. Such a method could either be part of a design tool, but could alternatively be part of a device having an adaptive filter. In that case the method enables that device to replace the polyphase components which it has calculated by an equivalent set of  
20 symmetrical modules.

These and other aspects of the invention are described in more detail with reference to the drawings. Therein:

- 25 Figure 1 illustrates a display system comprising a multirate filter according to the invention,

Figure 2 illustrates a mobile phone comprising a multirate filter according to the invention,

- 30 Figure 3A illustrates a filter in the form of a tapped delay line,  
Figure 3B illustrates a simplified version of the filter of Figure 3A,  
Figure 3C illustrates a filter in the form of an adding delay line,  
Figure 3D illustrates a simplified version of the filter of Figure 3C,  
Figure 4A shows a decimating filter,

Figure 4B shows implementation of the filter of Figure 4A in the form of a polyphase structure,

Figure 5A shows an interpolating filter,

Figure 5B shows implementation of the filter of Figure 5A in the form of a polyphase structure,

Figure 6A schematically shows a multirate filter according to the invention,

Figure 6B shows a first embodiment of the multirate filter according to the invention,

Figure 6C shows a second embodiment of the multirate filter according to the invention,

Figure 6D shows a third embodiment of the multirate filter according to the invention,

Figure 7A shows a filter having a rational decimation factor,

Figure 7B shows an implementation of this filter according to the invention.

FIG. 1 illustrates an example display system **a** having an image processor **b** that includes a multirate filter **e**. The filter **e** is implemented as a polyphase filter and allows both interpolation and decimation. Typically, the display system **a** includes filters for effecting both horizontal and vertical scaling. The controller **c** determines the appropriate scaling in each dimension, sets an appropriate mode of the filter **e** (interpolate or decimate, discrete phase or continuous phase, as required), and determines the appropriate coefficients that are provided by the memory **d**, depending upon the scaling and the mode.

Figure 2 shows another application of a multirate filter according to the invention, in this case as filter for eliminating noise and other interference from a received signal in a mobile telephone **h**. A signal carrying, for example, encoded speech is received by antenna **g**, downconverted to a baseband and amplified, if necessary, by receiver **h** then routed through an analog-to-digital converter (ADC) **i** for conversion to discrete samples. The discrete samples are routed through the multirate filter **j** which filters out frequencies below a selected threshold. The threshold is selected to distinguish between information components of the transmitted signal and noise and other interference components. The signals are then demodulated by a demodulator **k** and decoded by a signal decoder **l**. Output signals from the signal decoder **l** are routed to a speech decoder **m** for conversion to digitized voice signals. The digitized voice signals are converted to analog signals by a digital to

analog converter (DAC) for ultimate output through a speaker of the mobile telephone. Other components, such as error detection and correction components, may be provided as well within the receive portion of the mobile telephone.

Likewise the multirate filter according to the invention is applicable in the sending part of a mobile telephone. The architecture of the sending part (not shown) of a mobile telephone is globally the inverse of the receiving part. Hence, such a mobile telephone comprises an analog to digital converter for converting an analog speech signal into a digital speech signal. A speech encoder subsequently compresses the digital speech signal and provides the compressed signal. A signal encoder performs a channel encoding operation at the compressed signal and provides a channel signal. A modulator modulates the channel signal. The channel signal is filtered by a multirate filter according to the invention as described in more detail below. The filtered signal is then converted into an analog signal by a digital to analog converter. A transmitter transmits the analog signal.

In the present description a filter  $h[n]$  having z-transform  $H(z)$  is defined as  $(\alpha, \psi)$ -symmetric if it fulfills the following relationship:

$$(5) \quad H(z) = \alpha z^{-2\psi} H^*(z^{-1}), \text{ which corresponds to } h[n] = \alpha h^*[2\psi - n] \text{ in the time domain}$$

A special case is  $(\alpha=1, \psi)$ ,  $h[n]$  being a real-valued function. For these filters  $h[n]$  the following relation holds:

$$h[2\psi - i] = h[i] \text{ wherein } i \text{ is an integer,}$$

Examples are  $h[] = \langle a, b, c, b, a \rangle$  having symmetry  $(\alpha=1, \psi=2)$ , and  $h[] = \langle a, b, c, c, b, a \rangle$  having symmetry  $(\alpha=1, \psi=5/2)$

Another special case is  $(\alpha=-1, \psi)$ . For these filters either  $h[2\psi - i] = -h[i]$  wherein  $i$  is an integer,

These filters are also referred to as anti-symmetric filters, not to be confused with asymmetric filters.

Examples are:

$$h[] = \langle a, b, 0, -b, -a \rangle \text{ having symmetry } (\alpha=-1, \psi=2), \text{ and}$$

$$h[] = \langle a, b, 0, 0, -b, -a \rangle \text{ having symmetry } (\alpha=-1, \psi=5/2)$$

The sign of symmetry is however not limited to real values, as is illustrated by the following examples:

The filter  $h[] = \langle a+jb, b+ja \rangle$  having symmetry  $(\alpha=j, \psi=1/2)$ , and

$h[] = \langle a+jb, -b-ja \rangle$  having symmetry ( $\alpha=j, \psi=1/2$ ).

Filters having  $(\alpha, \psi)$ -symmetry as defined above can be implemented efficiently. It is known to exploit their symmetry by reducing the number of components. This is illustrated in Figures 3A-3D. Figure 3A shows a symmetric filter implemented in the form of a tapped delay line. In the example shown, the delay line comprises 4 delay elements  $z^{-1}$ , a first multiplier having multiplication factor  $a$  is coupled to the input of the delay line, a second, a third and a fourth multiplier for multiplying with multiplication factors  $b, c, d$  are coupled to subsequent connection nodes between the delay elements  $z^{-1}$ , and a fifth multiplier having multiplication factor  $e$  is coupled to the output of the delay line. The sum of the output signals of the five multipliers is the output signal of the filter. The filter of Figure 3A can be reduced to the filter of Figure 3B having only three multipliers if the multiplication coefficients  $a$  and  $e$  are equal on the one hand and the multiplication coefficients  $b$  and  $d$  are equal on the other hand.

Analogously Figure 3D shows an efficient implementation of the adding delay line of Figure 3C.

Another known technique in filter design is the decomposition into polyphase components. This is in particular relevant for decimating or interpolating filters, or filters using a combination of decimation and interpolation.

Any filter  $H(z)$  can be decomposed in  $R$  polyphase components  $H_{R,r}(z)$ , which are related to  $H(z)$  as:

$$H(z) = \sum_{r=0}^{R-1} z^{-r} H_{R,r}(z^R), \text{ wherein the polyphase components relate to the transfer function } h()$$

as:

$$H_{R,r}(z) = \sum_n h[Rn + r] z^{-n}$$

Such a polyphase decomposition of the filter has the advantage that the polyphase filter components can operate at a lower clock speed than a filter which is not decomposed.

This is illustrated in Figure 4A and 4B. Figure 4A shows a decimating filter  $H(z)$  with decimation factor  $D$ . The filter has to operate at the same clock speed as the signal  $X(z)$  processed by the filter. Figure 4B shows the decomposition of this filter into polyphase components  $H_{D,0}(z), H_{D,1}(z), \dots, H_{D,D-1}(z)$ . The polyphase components are coupled via a decimator to a delay line having delay elements with delay  $z^{-1}$ . The decimators decimate the

sampling frequency of the signals obtained from the delay line with a factor D, so that the polyphase components may operate now at a corresponding lower frequency.

Figure 5A and 5B illustrate a second example. In this case the output signal  $Y(z)$  is obtained from the input signal  $X(z)$  by an interpolating filter  $H(z)$  with interpolation factor I. In Figure 5A, the interpolated signal received by the filter  $H(z)$  has a frequency which is I times higher than the frequency  $X(z)$ . Figure 5B shows the filter of Figure 5A decomposed into its polyphase components  $H_{I:1-1}(z)$ ,  $H_{I:1-2}(z)$ , .....,  $H_{I:0}(z)$ . The polyphase components each receive the input signal  $X(z)$ , and may therefore operate at the same frequency as the signal  $X(z)$  instead of a frequency I times higher.

Unfortunately, even a symmetric filter may have asymmetric polyphase components, as is illustrated by the following example:

Consider the filter  $h[n]$  having symmetric impulse response  $\langle a, b, c, d, d, c, b, a \rangle$ . In case of multirate factor  $R=2$ , both its polyphase components  $h_{2:0}[n]$  and  $h_{2:1}[n]$  are non-symmetric:

$$h_{2:0}[n] = \langle a, c, d, b \rangle, \text{ and} \\ h_{2:1}[n] = \langle b, d, c, a \rangle.$$

It has been found by the inventors that the polyphase components of the multirate filter can be combined in  $(\alpha, \psi)$  symmetric filter modules. The general structure of such a multirate filter according to the invention is illustrated in Figure 6A.

The multirate filter according to the invention shown therein has:

- an input unit 10 for receiving an input signal  $S_{in}$  and for providing a plurality of intermediate signals IS1, IS2 in response to said input signal,
- a filter unit 20 coupled to the input unit 10,
- an output unit 30 coupled to the filter unit 20, for generating an output signal  $S_{out}$ .

The filter unit 20 comprises at least a first and a second filter module 21, 22, having a transfer function  $H_0(z)$  and a transfer function  $H_1(z)$  respectively, which are mutually related according to the relations

$$H_0(z) = c_0(H_B(z) + M_{\alpha, \psi} H_B(z)) \text{ and} \\ H_1(z) = c_1(H_B(z) - M_{\alpha, \psi} H_B(z)).$$

Therein  $M_{\alpha, \psi} H_B(z) = \alpha z^{-2\psi} H_B^*(z^{-1})$ , and  $H_B^*(z) = \sum h_b^*[m] z^{-m}$ .

$H_B(z)$  is the z-transform of  $h_b[m]$ .

By combining a non-symmetric filter  $H_B(z)$  with its mirrored version  $M_{\alpha,\psi} H_B(z)$ , either a symmetric filter (in case of combining by addition) or an anti-symmetric filter (in case of combining by subtraction) is obtained. This is illustrated by the following example. When adding the two non-symmetric polyphase components

$$h_{2:0}[n] = \langle a, c, d, b \rangle, \text{ and}$$

$$h_{2:1}[n] = \langle b, d, c, a \rangle,$$

the following symmetric transfer function is obtained:

$$h[n] = \langle a+b, c+d, d+c, b+a \rangle$$

Subtracting these polyphase components from each other gives the following anti-symmetric transfer function.

$$h'[n] = \langle a-b, c-d, d-c, b-a \rangle$$

The multirate filter comprises a combination unit 11 coupled to the filter modules 21, 22 for generating a first combination signal  $S_{sum}$  and a second combination signal  $S_{diff}$ . The combination unit 11 may be a part of the input unit 10, or a part of the output unit 30. Alternatively both the input unit 10 and the output unit 30 may comprise one or more combination units.

In an embodiment of the invention the basic transfer function  $H_B(z)$  is a polyphase component i.e.  $H_B(z) = H_{R:r}(z) = \sum_n h[Rn+r]z^{-n}$ .

Figure 6B shows a first subembodiment of said embodiment. Therein the combination unit 11 is comprised in the input unit 10. The input unit 10 generates a first and a second intermediate signal  $IS1$ ,  $IS2$  from the input signal  $S_{in}$ . The first and the second intermediate signal  $IS1$ ,  $IS2$  are delayed by delay functions  $z^{-rq}$  and  $z^{-sq}$  respectively, wherein  $rq$  and  $sq$  are integer numbers selected from  $Z$ . After the delay the signals are decimated with a factor  $D$ , so that the intermediate signals have a sampling rate which is  $D$  lower than that of the input signal  $S_{in}$ . The combination unit 11 subsequently generates the first and the second combination signal  $S_{sum}$ ,  $S_{diff}$  from the pair of intermediate signals  $IS1$ ,  $IS2$ . In the embodiment shown in Figure 6B the first combination signal  $S_{sum}$  is the sum of the intermediate signals  $IS1$  and  $IS2$  and the second combination signal  $S_{diff}$  is the difference of the intermediate signals  $IS1$  and  $IS2$ . The first filter module 21 filters the first combination signal  $S_{sum}$  and the second filter module 22 filters the second combination signal  $S_{diff}$ . The output unit 30 subsequently combines the output signals of the filter modules 21 and 22 to generate the overall output signal  $S_{out}$ . The output unit 30 may additionally perform an interpolation with a factor  $I$  and introduce an extra delay  $z^{-tp}$  to the output signal  $S_{out}$ .

As is clear from the above, the transfer functions of the filter modules 21 and 22 are symmetric. The transfer function  $H_0(z)$  of the first filter module is the sum of the basic transfer function  $H_B(z)$  and its mirrored counterpart, and the transfer function  $H_1(z)$  of the second filter module is the difference of the basic transfer function  $H_B(z)$  and its mirrored counterpart. Hence, both transfer functions  $H_0(z)$  and  $H_1(z)$  allow an efficient implementation, having a relatively small number of multipliers. On the one hand the filter modules can operate relatively slowly as they process the signal  $S_{sum}$  and  $S_{diff}$ , which are derived from the decimated signals  $IS1$  and  $IS2$ . The optional interpolation stage only takes place after the filter unit 20.

Figure 6C shows a second subembodiment wherein the transfer function  $H_B(z)$  is a polyphase component. In this case the first and the second filter module 121, 122 each filter a respective one of the intermediate signals  $IS1$ ,  $IS2$ . The combination unit 131 is comprised in the output unit 130 and generates the first combination signal  $S_{sum}$  and the second combination signal  $S_{diff}$  from output signals  $OS1$ ,  $OS2$  of the first and the second filter module. Optionally the combination signals are interpolated by interpolation factor  $I$  and delayed by a delay  $z^{-\tau_p}$  and  $z^{-\tau_p}$  respectively before they are recombined into the output signal. Likewise in this case the filter modules 121 and 122 are symmetric, and can be implemented with relatively small number of multiplier. Because the filter is operative after decimation and before interpolation, the filter can operate at a relatively low clock speed.

The transfer functions  $H_0(z)$  and  $H_1(z)$  of the first and the second filter module are the sum of a basic transfer function  $H_B(z)$  and its mirrored version  $M_{\alpha,\psi}H_B(z)$ . In the embodiments shown in Figure 6B and 6C the basic transfer function is a polyphase component of the transfer function of the multirate filter.

Figure 6D shows an embodiment wherein the basic transfer function  $H_B(z)$  from which the transfer functions  $H_0(z)$  and  $H_1(z)$  are composed is derived from two polyphase components according to the relation:

$$H_B(z) = H_{R,\tau_0}(z) + z^b H_{R,\tau_1}(z).$$

The combination unit 211 is comprised in the input unit 210, and generates the first and the second combination signal  $S_{sum}$ ,  $S_{diff}$  from a pair of the said intermediate signals  $IS1$ ,  $IS2$ . The first filter module 221 filters the first combination signal  $S_{sum}$  and the second filter module 222 filters the second combination signal  $S_{diff}$ . The filter unit 220 further comprises a third filter module 223 with a transfer function  $H_2(z)$  and a fourth filter



module 224 with a transfer function  $H_3(z)$  which are mutually related according to the relations

$$H_2(z) = c_2(H'_B(z) - MH'_B(z))$$

$$H_3(z) = c_3(H'_B(z) + MH'_B(z)).$$

5 Also the further basic transfer function  $H_B'(z)$  is derived from a combination of two polyphase components:

$$H'_B(z) = H_{R:r_0}(z) - z^b H_{R:r_1}(z)$$

The third 223 and fourth filter module 224 each filter a respective one of the first combination signal Ssum and the second combination signal Sdiff. The filter comprises a  
10 first further combination unit 231 for generating a first auxiliary signal AS1 representing the sum of the output signals of the first filter module 221 and the second filter module 222, and a second auxiliary signal AS2 representing the sum of the output signals of the third filter module 222 and the fourth filter module 223. The filter further comprises a second further combination unit 232 for generating a further sum signal Ssum'' and a further difference  
15 signal Sdiff'' from the first and the second auxiliary signal AS1, AS2. These signals are recombined into an output signal Sout after an optional interpolation with a factor I. The signals Ssum'' and Sdiff'' may further be delayed with an additional delay  $z^{-r_0P}$  and  $z^{-r_1P}$ .

As in the embodiments shown in Figures 6B and 6C the filter unit 220 of the multirate filter can on the one hand operate at a relatively low speed, as compared to a filter  
20 without polyphase decomposition. And on the other hand the modules 221, 222, 223, 224 of the filter unit 220 have a symmetric transfer function, so that they can be implemented with a relatively small number of multipliers.

Figure 7A and 7B illustrate a further embodiment of the invention. Figure 7A shows a multirate filter having a rational decimation factor 3/2.

25 Figure 7B shows the inventive implementation of this filter. The filter unit 320 comprises two groups of filtermodules. One group comprises the modules 322, 324, 325 and 326, having transfer functions  $E_0(z)$ ,  $E_1(z)$ ,  $E_2(z)$  and  $E_3(z)$  respectively. The other group comprises the modules 321 and 322 having transfer functions  $E_4(z)$  and  $E_5(z)$  respectively. The transfer functions for the first group are derived from the polyphase components of the  
30 original filter  $H(z)$  as follows.

$$E_0(z) = \frac{1}{4}(H_{6:0}(z) + H_{6:-2}(z) + H_{6:3}(z) + H_{6:1}(z))$$

$$E_1(z) = \frac{1}{4}(H_{6:0}(z) + H_{6:-2}(z) - H_{6:3}(z) - H_{6:1}(z))$$

$$E_2(z) = \frac{1}{4}(H_{6:0}(z) - H_{6:-2}(z) + H_{6:3}(z) - H_{6:1}(z))$$

$$E_3(z) = \frac{1}{4}(H_{6:0}(z) - H_{6:-2}(z) - H_{6:3}(z) + H_{6:1}(z)).$$

5 The transfer functions for the second group are

$$E_4(z) = \frac{1}{2}(H_{6:2}(z) + H_{6:5}(z))$$

$$E_5(z) = \frac{1}{2}(H_{6:2}(z) - H_{6:5}(z)).$$

10 It is noted that the skilled person will consider many alternatives within the scope of the invention as defined by the claims. For example the delay functions  $z^2$ ,  $z^0$  and  $z^1$  in the input unit may be replaced by delay functions  $z^{2-k}$ ,  $z^{-k}$  and  $z^{1-k}$  in combination with a delay function  $z^k$  in the signal line providing the input signal  $X(z)$ . It is noted that merely the relative delays generated between the signals provided by the delay functions in the input unit 310 is of importance.

15 It is also clear to the skilled person that several functions may be interchanged. For example, the decimate units indicated by the downpointing arrows with label 3 may be located at the outputs of the combination unit 311. Furthermore the delay units indicated with  $z^0$  and  $z^1$  may be included in the combination unit 311. In that case a first combination element of the combination unit 311 (the upper one in the drawing) generates the signal  $X(z)$   
 20  $+ z^1 X(z) = X(z)(1+z)$ . The other combination element of said combination unit generates  $X(z)(1-z)$ .

The skilled person will further realized that other operations can be interchanged for example the sequence of operations:

25 interpolating with a factor I,  
 delaying with  $z^a$ ,  
 decimating with a factor D

may be replaced by the sequence

30 delaying with  $z^{aq}$ ,  
 decimating with a factor D  
 interpolating with a factor I,

delaying with  $z^{ap}$ .

Provided that D and I are coprime.

Other examples are that multiplication operations can be interchanged with delay operations and with decimation or interpolation operations.

5 A multirate filter according to the invention can be designed by carrying out the following steps:

a. decomposing a symmetrical filter H into its polyphase components  $H_{R:1}$ , ...,  $H_{R:R-1}$ .

This kind of decomposition is well known, and is described for example in the thesis  
10 "Efficiency in multirate and complex digital signal processing" by A.W.M. van den Enden, ISBN 90 6674 650 5. In particular reference is made to chapter 2.8 and to Appendix C thereof.

The so obtained set of polyphase components may be replaced by a set of symmetrical modules as follows. Naturally those polyphase components which are already symmetrical  
15 need not be replaced. The following situations may occur.

b. The set of polyphase components may comprise pairs of components  $H_{R:r1}$  and  $H_{R:r2}$  which are asymmetrical, which share a common input, and which are related to each other according to the relation  $H_{R:r2} = M_{\alpha,\psi}(H_{R:r1})$ . The parameters  $\alpha, \psi$  can be different for each pair. Such pairs of components  $H_{R:r1}$  and  $H_{R:r2}$  can be replaced by

20 a first module with transfer function  $H_0 = H_{R:r1} + H_{R:r2}$ ,

a second module with transfer function  $H_1 = H_{R:r1} - H_{R:r2}$ , wherein the first and the second module sharing the common input, and by

a combination unit for generating a first and a second combination signal from the output signals of the first and the second module.

25 c. The set of polyphase components may comprise pairs of components  $H_{R:r1}$  and  $H_{R:r2}$  which are asymmetrical, which share a common output, and which are related to each other according to the relation  $H_{R:r2} = M_{\alpha,\psi}(H_{R:r1})$ , wherein the parameters  $\alpha, \psi$  can be different for each pair.

Each such a pair of components  $H_{R:r1}$  and  $H_{R:r2}$  can be replaced by

30 a first module with transfer function  $H_0 = H_{R:r1} + H_{R:r2}$ ,

a second module with transfer function  $H_1 = H_{R:r1} - H_{R:r2}$ , which modules share the common output, and by

a combination unit for generating a first combination signal from a first and a

second intermediate signal, and providing said combination signal to the third module, and for generating a second combination signal from a first and a second intermediate signal, and providing said combination signal to the fourth module.

d. Finally components may occur in quadruplets which are asymmetrical and which comprise

a first pair  $H_{R:r1}$ ,  $H_{R:r2}$  which are related by  $H_{R:r2} = M_{\alpha,\psi}(H_{R:r1})$

a second pair  $H_{R:r3}$ ,  $H_{R:r4}$  which are related by  $H_{R:r4} = M_{\alpha,\psi}(H_{R:r3})$

wherein,

the components  $H_{R:r1}$  and  $H_{R:r3}$  share a first common input,

the components  $H_{R:r2}$  and  $H_{R:r4}$  share a second common input,

the components  $H_{R:r1}$  and  $H_{R:r2}$  share a first common output, and

the components  $H_{R:r3}$  and  $H_{R:r4}$  share a second common output.

Each such a quadruplet can be replaced by a quadruplet of symmetrical filter modules and three combination units. The quadruplet of filter modules comprises

a first module having transfer function  $H_0 = H_{R:r0} + H_{R:r1} + H_{R:r2} + H_{R:r3}$ ,

a second module having transfer function  $H_1 = H_{R:r0} - H_{R:r1} - H_{R:r2} + H_{R:r3}$ ,

a third module having transfer function  $H_2 = H_{R:r0} + H_{R:r1} - H_{R:r2} - H_{R:r3}$ ,

a fourth module having transfer function  $H_3 = H_{R:r0} - H_{R:r1} + H_{R:r2} - H_{R:r3}$ .

A first combination unit generates a first combination signal from input signals received at the first and the second common input, and provides said first combination signal to the first and the second module. It also generates a second combination signal from those input signals, and provides that to the third and the fourth module.

A second combination unit generates a first auxiliary signal from output signals generated by the first and the third unit. It generates a second auxiliary signal from output signals generated by the second and the fourth unit.

A third combination unit generates a first and a second output signal from the first and the second auxiliary signal.

It is noted that the situations b,c and d may occur in combinations.